

# Inflation and primordial power spectra at anisotropic spacetime inspired by Planck's constraints on isotropy of CMB

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## Abstract

The recently released Planck 2013 results show that the primordial fluctuations are deviated from isotropy. The deviations from isotropy are robust against component separation algorithm, mask and frequency dependence. To incorporate the Planck's data into standard cosmological model, we propose an inflation of the very early universe in an anisotropic spacetime. A generalized Friedmann-Robertson-Walker (FRW) metric is presented in the Randers-Finsler spacetime. By employing the osculating Riemannian approach, we obtain the primordial power spectra of the scalar perturbation with direction dependence, such as the dipolar modulation. This is consistent with the deviations from isotropy of the universe found by the Planck satellite.

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## I. INTRODUCTION

The cosmological principle assumes that the universe is statistically isotropic and homogeneous at large scales. Recently, this principle has been rigorously tested by the CMB anisotropy observations from the Planck satellite [1]. The Planck satellite [2] has found deviations from isotropy (around  $3\sigma$ ). Several anomalies of the CMB anisotropy have been confirmed, such as the quadrupole-octopole alignment [3], the hemispherical asymmetry [4, 5], the parity asymmetry [6–12], and etc.. Besides the Planck’s results, the Wilkinson Microwave Anisotropy Probe (WMAP) [13] has previously found evidence for the deviations from isotropy and the anomalies. Via dealing with the WMAP dataset, the level of statistical anisotropy was constrained to be  $g \sim 0.15$  in the primordial power spectra of the form  $\mathcal{P}(\mathbf{k}) = \mathcal{P}(k)(1 + g(\mathbf{k} \cdot \mathbf{n})^2)$  where  $\mathbf{n}$  denotes a privileged direction in the space [14–16]. These anomalies might stem from a non-standard spinor [17] or certain primordial vector fields [18–27].

To incorporate the observed deviations from isotropy in the Planck data into the standard cosmological model, we should try to investigate properties of inflation at an anisotropic spacetime. The natural framework of anisotropic spacetime is the so-called Finsler geometry [28]. Finsler geometry gets rid of the quadratic constraint on the metric [29–31]. The Finsler spacetime admits certain privileged axes and permits less symmetries than the Riemann one [32–34]. Thus, the Finsler geometry is a reasonable candidate to reveal the deviations from isotropy of the spacetime. For instance, the Finsler spacetime could account for the Lorentz violation as well as the CPT violation [35–43]. The cosmic acceleration could be explained by the anisotropic Friedmann equation in Finsler cosmology [44]. The large-scale bulk flow [45, 46] could be revealed by a Finslerian Zermelo navigation model [47]. The Randers structure could account for the privileged direction [48–50] of the maximum accelerating expansion in the Hubble diagram [51]. A spatially anisotropic Finslerian model could account for the mass discrepancy problem of the Bullet cluster [53], and etc..

The primordial vector fields may be related with the Randers spacetime. The Randers structure [54] involves an extra 1-form which is related to a vector field. This vector field may influence the very early evolution of the universe if it existed in the universe. In this paper, we propose a generalized FRW metric in the Randers spacetime. The Randers structure comprises the FRW part and the weak vector field. The vector field singles out a privileged

axis in the very early universe. We study Einstein's gravitational field equations with the osculating Riemannian approach. The time-time component of Einstein's field equations is resolved to obtain an inflationary phase of the very early universe. The anisotropic modifications are studied for the inflationary universe. By studying the equation of motion for the inflaton field, we obtain the primordial power spectra of the scalar perturbation with statistical anisotropy. The predicted statistical anisotropy may account for the Planck observed deviations of isotropy and certain anomalies in the CMB anisotropy.

The rest of the paper is arranged as follows: In section II, we first present a brief introduction to Finsler geometry. In section III, the osculating Riemannian approach is used to study a generalized FRW metric in the Randers spacetime. The geometric objects are obtained. In section IV, we study the time-time component of Einstein's gravitational field equations and resolve it to obtain an inflationary solution. Meanwhile, the anisotropic modifications are acquired for the inflationary phase of the universe. The primordial power spectra with statistical anisotropy is got through the equation of motion for the inflaton field in section V. Conclusions and discussions are listed in section VI.

## II. THE ANISOTROPIC SPACETIME

In this paper, we suggest the Randers spacetime as a suitable background of inflation at the very early stage of the universe. The Randers spacetime is a class of Finsler spacetimes. Thus, we first present a brief introduction to the Finsler geometry in this section. For more detailed discussions on Finsler geometry, see the references, for instance [29–31].

Different from Riemann geometry, Finsler geometry is defined on the tangent bundle. Finsler space stems from the arc-length integrals of the form

$$s = \int_a^b F(x, y) d\tau , \quad (1)$$

where  $x$  and  $y \equiv dx/d\tau$  denote the location and the velocity, respectively. The proper length is given by  $\tau$  (or  $s$ ). The integrand  $F(x, y)$  is called the Finsler structure. It is a smooth and positive function on the tangent bundle. It is positively homogeneous of degree one, i.e.,  $F(x, \lambda y) = \lambda F(x, y)$  for  $\lambda > 0$ . The Finsler metric is defined by the Hessian,

$$g_{\mu\nu} = \frac{\partial}{\partial y^\mu} \frac{\partial}{\partial y^\nu} \left( \frac{1}{2} F^2 \right) . \quad (2)$$

Together with its inverse  $g^{\mu\nu}$ , it is used to lower and raise the spatial indices of tensors. For the Finsler spacetime, the spatial indices run from 1 to 3 and the temporal index runs 0 in this paper.

The Finsler metric is dependent on the positions  $x$  as well as the directions given by the fibre  $y$ . This is different from the Riemann metric which depends on  $x$  only. Thus, there is other significant object, i.e., the Cartan (torsion) tensor in Finsler geometry. The Cartan tensor is defined as [30]

$$C_{\mu\nu\sigma} = \frac{1}{2} \frac{\partial g_{\mu\nu}}{\partial y^\sigma} . \quad (3)$$

It completely characterizes the deviation of the Finsler space from the Riemann space. The Finsler space becomes Riemannian if and only if  $C_{\mu\nu\sigma} = 0$ . This result reveals that Finsler geometry is a natural generalization of Riemann geometry.

The Randers space [54] is a kind of Finsler spaces. It could be viewed as a Riemann space influenced by a primordial vector field (for instance, the electromagnetic field). The Randers structure is given by

$$F(x, y) = \alpha(x, y) + \beta(x, y) , \quad (4)$$

where  $\alpha(x, y) = \sqrt{\tilde{a}_{\mu\nu}(x)y^\mu y^\nu}$  denotes a Riemann structure and  $\beta(x, y) = \tilde{b}_\mu(x)y^\mu$  is a 1-form. However, the vector field appears as a part of the space structure in the Randers space. Furthermore, it induces the anisotropic properties of the Randers space. The tildes here denote raising (or lowering) the indices with the Riemann metric  $\tilde{a}_{\mu\nu}$  (or  $\tilde{a}^{\mu\nu}$ ). To reveal the anisotropy of the Randers space, one notices that the Randers structure is not absolutely homogeneous of degree one. Otherwise, the Randers structure reduces back to the Riemann one. The reason is that  $\alpha$  would keep invariant while  $\beta$  changes its sign under the rescale  $y \rightarrow -y$ . In addition, the anisotropy of the Finsler spaces could also be revealed by the number of solutions of the Killing equations [32–34].

In the Randers space, the Randers metric is defined as [30]

$$g_{\mu\nu} = \frac{F}{\alpha} \left( \tilde{a}_{\mu\nu} - \tilde{\ell}_\mu \tilde{\ell}_\nu \right) + \ell_\mu \ell_\nu , \quad (5)$$

where  $\ell_\mu = \tilde{\ell}_\mu + \tilde{b}_\mu$  and  $\tilde{\ell}_\mu = \tilde{a}_{\mu\nu} y^\nu / \alpha$ . Its inverse is given by

$$g^{\mu\nu} = \frac{\alpha}{F} \tilde{a}^{\mu\nu} + \frac{\alpha^2}{F^2} \frac{\beta + \alpha \|\tilde{b}\|^2}{F} \tilde{\ell}^\mu \tilde{\ell}^\nu - \frac{\alpha^2}{F^2} \left( \tilde{\ell}^\mu \tilde{b}^\nu + \tilde{\ell}^\nu \tilde{b}^\mu \right) , \quad (6)$$

where  $\|b\|^2 = \tilde{b}^\mu \tilde{b}_\mu$ . The Cartan tensor of Randers space is given as [30]

$$C_{\mu\nu\sigma} = \frac{1}{2\alpha} \mathcal{S}_{(\mu\nu\sigma)} \left( \tilde{a}_{\mu\nu} - \tilde{\ell}_\mu \tilde{\ell}_\nu \right) \left( \tilde{b}_\sigma - \frac{\beta}{\alpha} \tilde{\ell}_\sigma \right), \quad (7)$$

where  $\mathcal{S}_{(\mu\nu\sigma)}$  refers the summation over the cyclic permutation of indices. We note that each term in  $C_{\mu\nu\sigma}$  is proportional to the components of  $\tilde{b}$ . Thus, the Cartan tensor could be a first order object in the case of  $\|\tilde{b}\| \ll 1$ .

### III. THE OSCULATING RIEMANNIAN APPROACH

In this section, we consider a generalized FRW metric with weak anisotropy in the Randers spacetime. Following Stavrinou *et al.*'s approach [57], we will employ the osculating Riemannian metric to approximate the Randers metric.

The related Randers structure is given by the spatially flat FRW metric  $\tilde{a}_{\mu\nu} = (1, -a^2(t), -a^2(t), -a^2(t))$  and an extra 1-form  $\tilde{b}_\mu dx^\mu$  (or a vector field  $\tilde{b}^\mu \partial/\partial x^\mu$ ). The temporal coordinate denotes the proper time measured by a comoving observer while the spatial coordinates are comoving. The velocity  $y^\mu = (1, 0, 0, 0)$  denotes the tangent 4-velocity of the comoving observer along a worldline. Here  $\tau$  denotes the proper time. The vector field is very weak, i.e.,  $\|\tilde{b}\| \ll 1$  in the Randers spacetime. It would be expected to align with the tangent 4-velocity [55]. Thus, we consider the timelike vector field that has only the temporal component non-vanishing, i.e.,  $\tilde{b}_\mu = (B(z), 0, 0, 0)$ . In addition, the component  $B(z)$  is set to be dependent on the third spatial coordinate  $z$  only. This proposition would induce the anisotropy of the primordial power spectra along the  $z$ -axis. Therefore, we will study the generalized FRW metric (i.e., the Randers structure) as

$$d\tau = \sqrt{dt^2 - a^2(t) (dx^2 + dy^2 + dz^2)} + B(z)dt, \quad (8)$$

where the spatially flat FRW part is given by the first term on the right hand side. The anisotropy of the obtained Randers spacetime is characterized by the second term on the right hand side of the above equation. For the weak vector field, the Randers metric could be viewed as a slight modification to the FRW metric.

Usually, it is difficult to discuss the gravitational issues in the completely Finsler geometric framework. However, the Finsler metric could be approximately related to the Riemann metric in certain cases [29, 56]. Actually, it is convenient to study the gravity with the

osculating Riemannian method [29]. The Finsler structure corresponds to the osculating Riemannian metric as

$$g_{\mu\nu}(x) \equiv g_{\mu\nu}(x, y(x)) , \quad (9)$$

where the velocity  $y(x)$  is viewed as a function of the position  $x$ . Then the osculating Riemannian structure becomes  $ds^2 = (1 + B(z))^2 dt^2 - a^2(t) (1 + B(z)) (dx^2 + dy^2 + dz^2)$  for the above generalized FRW metric (8) in the Randers spacetime. Correspondingly, the Christoffel symbols are defined as [57]

$$\gamma_{\nu\sigma}^\mu(x) = \gamma_{\nu\sigma}^\mu(x, y(x)) + C_{\nu\lambda}^\mu(x, y(x)) \frac{\partial y^\lambda}{\partial x^\sigma}(x) + C_{\lambda\sigma}^\mu(x, y(x)) \frac{\partial y^\lambda}{\partial x^\nu}(x) - g^{\mu\rho}(x, y(x)) C_{\kappa\nu\sigma}(x, y(x)) \frac{\partial y^\kappa}{\partial x^\rho}(x) , \quad (10)$$

where  $\gamma_{\nu\sigma}^\mu(x, y(x)) = \frac{g^{\mu\kappa}}{2} \left( \frac{\partial g_{\kappa\sigma}}{\partial x^\nu} - \frac{\partial g_{\nu\sigma}}{\partial x^\kappa} + \frac{\partial g_{\nu\kappa}}{\partial x^\sigma} \right)$ . From the equation (7), we may note that the Cartan tensor is proportional to the components of the vector  $\tilde{b}$  which is small. In addition, we have chosen the comoving coordinates in the Randers spacetime. Thus, the terms dependent on the Cartan tensor could be dropped in the equation (10). In the Randers spacetime, therefore, we approximately obtain the Christoffel symbols as

$$\Gamma_{\nu\sigma}^\mu(x) \approx \gamma_{\nu\sigma}^\mu(x, y(x)) . \quad (11)$$

Here  $\Gamma_{\nu\sigma}^\mu(x)$  denote the coefficients of the osculating Christoffel connection.

Corresponding to the osculating Christoffel connection, the components of the curvature tensor are given by

$$R_{\nu}{}^\mu{}_{\rho\sigma} = \Gamma_{\nu\sigma,\rho}^\mu - \Gamma_{\nu\rho,\sigma}^\mu + \Gamma_{\nu\sigma}^\kappa \Gamma_{\kappa\rho}^\mu - \Gamma_{\nu\rho}^\kappa \Gamma_{\kappa\sigma}^\mu . \quad (12)$$

The Ricci tensor and the scalar curvature are, respectively, given as

$$Ric_{\mu\nu} = R_{\mu}{}^\kappa{}_{\kappa\nu} , \quad (13)$$

$$S = g^{\mu\nu}(x) Ric_{\mu\nu} . \quad (14)$$

The osculating Riemannian metric evolves following the conventional Einstein's gravitational field equations,

$$Ric_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S = 8\pi G T_{\mu\nu} , \quad (15)$$

where  $T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left( \frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - V(\phi) \right)$  denotes the energy-momentum tensor of the cosmic fluid. However, the anisotropic properties of the Finsler spacetime have been approximately comprised into the osculating Riemannian metric. Thus, the obtained Einstein's field equations are weakly anisotropic in the Randers spacetime considered in this paper.

#### IV. THE INFLATIONARY PHASE WITH WEAK ANISOTROPY

In this section, we study Einstein's gravitational field equations in the Randers spacetime. The inflationary solution is obtained for the very early universe at the zero-order assumption. Further, we take account the slight modifications to the inflationary phase such that the very early spacetime acquires the weak anisotropy.

First, we calculate the time-time component of the Einstein tensor to study Einstein's field equations in the Randers spacetime. The Einstein tensor is defined as  $G_{\mu\nu} = Ric_{\mu\nu} - \frac{1}{2}g_{\mu\nu}S$ . We obtain its time-time component as

$$G_{00} = 3 \left( \frac{\dot{a}}{a} \right)^2 + \left( \frac{1}{a} \right)^2 \frac{3B'^2 - 4B''(1+B)}{4(1+B)}, \quad (16)$$

where the dots denote the derivative of  $a$  with respect to  $t$  while the primes denote the derivative of  $B$  with respect to  $z$ . For an inflationary phase [58–62], the very early universe undergoes a process of exponential expansion, i.e.,  $a \sim e^{Ht}$  where  $H$  is the Hubble horizon. Thus, the second term on the right hand side of the equation (16) would decrease exponentially. It could be discarded at the zero-order approximation. In this way, Einstein's field equation would reduce back to the conventional one. To account for the spacetime anisotropy, we could consider the modifications from the second term on the right hand side of (16).

At the zero-order approximation, the time-time component of the Einstein tensor becomes

$$G_{00}^{(0)} = 3 (\dot{a}/a)^2. \quad (17)$$

On the other hand, the energy-momentum tensor of the cosmic fluid is characterized by the one of the perfect fluid. At the zero order, its time-time component is given by [63]

$$T_{00}^{(0)} = \rho^{(0)} = \frac{1}{2} \left( \frac{d\phi^{(0)}}{dt} \right)^2 + V^{(0)}(\phi^{(0)}), \quad (18)$$

where  $\phi^{(0)}(t)$  is the zero-order part of the mostly homogeneous inflaton field  $\phi(t, \vec{x})$ . Consider the slow-roll condition that the inflaton field slowly rolls down its potential, i.e.,  $\left( \frac{d\phi^{(0)}}{dt} \right)^2 \ll V^{(0)}(\phi^{(0)})$ . Thus, the energy density  $T_{00}^{(0)}$  of the inflaton field almost becomes a constant. The time-time component of Einstein's equations becomes [63]

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} V^{(0)}(\phi^{(0)}) = \text{const.}, \quad (19)$$

which determines the evolution of scale factor  $a$ . This equation has an exponential solution  $a(t) \sim e^{Ht}$  where  $H$  denotes the Hubble horizon which is almost constant. The exponential expansion of the very early universe corresponds to the inflationary phase of the universe. Note that the above analysis is similar to that in the standard inflationary model, see [63] for details.

To account for the anisotropic effects, we substitute the exponential solution  $a(t) \sim e^{Ht}$  back into Einstein's field equation (15) and obtain the anisotropic modification  $B(z)$  of the very early universe. In this case, the time-time component of the Einstein tensor is given by the equation (16) the energy density of the inflaton field is given by  $T_{00} = \frac{1}{2} \left( \frac{d\phi^{(0)}}{dt} \right)^2 + (1+B)^2 V(\phi^{(0)})$ . Here the potential of the inflaton field is set to be  $V(\phi^{(0)}) = V^{(0)}(\phi^{(0)}) / (1+B)^2$ , where the correction  $(1+B)^2$  in  $T_{00}$  comes from the osculating Riemannian metric  $g_{00}$ . Then the time-time component of Einstein's field equation (15) becomes

$$3B'^2 - 4B''(1+B) = 0, \quad (20)$$

where we have used the equations (16), (17), (18) and (19) in calculations. We see that the remained term in the equation (20) determines the anisotropic modifications to the inflationary phase of the universe. The equation (20) has a solution as

$$B(z) = \frac{1}{256} (c_1 c_2)^4 \left( 1 + \frac{z}{c_2} \right)^4 - 1, \quad (21)$$

where  $c_1$  and  $c_2$  are the integral constants. In addition,  $c_2 > 0$  denotes a given distance scale which could be constrained by cosmological observations. We could set  $|c_1 c_2| \approx 4$  and  $|z| \ll c_2$  because of the condition  $|B(z)| \ll 1$ . Note that  $B(z)$  would be a monotonically increasing function of  $z$ . To the second-order approximation, the above solution  $B(z)$  could be expanded as  $B(z) \simeq f_0 + f_1 z + \frac{3f_1^2}{8(1+f_0)^2} z^2 + o(z^2)$  where  $f_0 = |c_1 c_2| - 4$  and  $f_1 = 4(1+f_0)/c_2$ . In this way, we obtain the inflationary phase of the generalized FRW metric (8) with weak anisotropy in the Randers spacetime.



## V. THE PRIMORDIAL POWER SPECTRA WITH THE DIRECTION DEPENDENCE

In this section, we study the primordial power spectra [64–70] with weak anisotropy in the Randers spacetime. First, the action of the inflaton field is given by

$$S = \int d^4x \sqrt{-g} \mathcal{L} = \int d^4x \sqrt{-g} \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) \quad (22)$$

where  $\sqrt{-g} = \det(g_{\mu\nu})$  for the osculating Riemannian metric of the Randers structure (8). From the Euler-Lagrange equation  $\sqrt{-g} \frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left( \sqrt{-g} \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} \right) = 0$ , we obtain the equation of motion of the inflaton field as

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1+B}{a^2} \nabla^2 \phi + \frac{dV^{(0)}(\phi)}{d\phi} - \frac{3}{2} \frac{B'}{a^2} \phi' = 0. \quad (23)$$

We have decomposed the inflaton field into two parts  $\phi(t, \mathbf{x}) = \phi^{(0)}(t) + \delta\phi(t, \mathbf{x})$ , where  $\delta\phi(t, \mathbf{x})$  denotes a first-order perturbation.

For the zero-order part  $\phi^{(0)}(t)$ , the equation of motion becomes

$$\ddot{\phi}^{(0)} + 3H\dot{\phi}^{(0)} + \frac{dV^{(0)}(\phi^{(0)})}{d\phi^{(0)}} = 0. \quad (24)$$

For the anisotropic and inhomogeneous perturbation, the equation of motion for the fluctuations becomes

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} + \frac{d^2V^{(0)}}{d\phi^2} \Big|_{\phi=\phi^{(0)}} \delta\phi - \frac{1+B}{a^2} \nabla^2 \delta\phi - \frac{3}{2} \frac{B'}{a^2} \delta\phi' = 0, \quad (25)$$

where we have used the equation (24) to eliminate  $\phi^{(0)}$ . Typically, the  $\frac{d^2V^{(0)}}{d\phi^2}$  term is small, which is proportional to the slow-roll variables. Thus, it could be neglected in the following discussions.

The fluctuation of inflaton field could be expanded into the Fourier modes as

$$\delta\phi(t, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} \delta\phi_{\mathbf{k}}(t). \quad (26)$$

In this way, the fluctuations would evolve along the equation of motion as

$$\ddot{\delta\phi}_{\mathbf{k}} + 3H\dot{\delta\phi}_{\mathbf{k}} + k_{eff}^2 \delta\phi_{\mathbf{k}} = 0, \quad (27)$$

where the effective wavenumber is given as

$$k_{eff}^2 = k^2 \left( 1 + B - i \frac{3B'}{2k} (\hat{\mathbf{k}} \cdot \hat{\mathbf{n}}) \right), \quad (28)$$

and  $\hat{\mathbf{k}} \cdot \hat{\mathbf{n}}$  denotes the cosine of angle between the wavevector  $\mathbf{k}$  and the third spatial direction  $\hat{\mathbf{n}}$ .

With the redefinition of the inflaton  $\delta\phi_{\mathbf{k}} = \delta\sigma_{\mathbf{k}}/a$ , we could work in the conformal time  $d\tau = dt/a$  where the coefficient  $(1+B)^2$  is discarded since it does not affect the following discussions. Thus, the equation of motion of the fluctuations becomes

$$\frac{d^2\delta\sigma_{\mathbf{k}}}{d\tau^2} + \left(k_{eff}^2 - \frac{1}{a} \frac{d^2a}{d\tau^2}\right) \delta\sigma_{\mathbf{k}} = 0 . \quad (29)$$

For the inflationary phase of the universe, the scale factor expands nearly exponentially, i.e.,  $a(t) \sim e^{Ht}$ . Correspondingly, the conformal scale factor reads

$$a(\tau) = -\frac{1}{H\tau} \quad (\tau < 0) . \quad (30)$$

Thus, we find that the equation becomes as

$$\frac{d^2\delta\sigma_{\mathbf{k}}}{d\tau^2} + \left(k_{eff}^2 - \frac{2}{\tau^2}\right) \delta\sigma_{\mathbf{k}} = 0 . \quad (31)$$

This equation has an exact solution [63]

$$\delta\sigma_{\mathbf{k}} = \frac{e^{ik_{eff}\tau}}{\sqrt{2k_{eff}}} \left(1 + \frac{i}{k_{eff}\tau}\right) . \quad (32)$$

By considering the redefinition of  $\delta\phi_{\mathbf{k}}$ , we obtain

$$\begin{aligned} |\delta\phi_{\mathbf{k}}|^2 &= \frac{1}{a^2} \frac{1}{2|k_{eff}|} \left(1 + \frac{1}{(|k_{eff}|\tau)^2}\right) \\ &= \frac{H^2\tau^2}{2|k_{eff}|} \left(1 + \frac{1}{(|k_{eff}|\tau)^2}\right) . \end{aligned} \quad (33)$$

The primordial power spectra of  $\delta\phi_{\mathbf{k}}$ , which is denoted by  $\mathcal{P}_{\delta\phi_{\mathbf{k}}}$ , is defined by

$$<0|\delta\phi_{\mathbf{k}}^*\delta\phi_{\mathbf{k}'}|0> = \delta^{(3)}(\mathbf{k} - \mathbf{k}') \frac{2\pi^2}{k^3} \mathcal{P}_{\delta\phi_{\mathbf{k}}} . \quad (34)$$

Thus, one has the primordial power spectra of  $\delta\phi_{\mathbf{k}}$  as

$$\begin{aligned} \mathcal{P}_{\delta\phi_{\mathbf{k}}} &= \frac{k^3}{2\pi^2} |\delta\phi_{\mathbf{k}}|^2 \\ &= \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{|k_{eff}|}\right)^3 + \left(\frac{H}{2\pi}\right)^2 \frac{k}{|k_{eff}|} (k\tau)^2 . \end{aligned} \quad (35)$$

For the super-horizon perturbations, the wavelength  $k$  is much larger than the horizon, i.e.,  $k\tau \ll 1$ . Thus, the primordial power spectra of  $\delta\phi_{\mathbf{k}}$  approximately becomes

$$\mathcal{P}_{\delta\phi_{\mathbf{k}}} = \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{|k_{eff}|}\right)^3 . \quad (36)$$

Note that  $|k_{eff}|^2 = k^2 \left( (1+B)^2 + (3B'/2k)^2 (\hat{\mathbf{k}} \cdot \hat{\mathbf{n}})^2 \right)$  contains the anisotropic properties.

On the super-horizon scales, the primordial power spectra of the comoving curvature perturbation  $\mathcal{R}$  is given by [71]

$$\mathcal{P}_{\mathcal{R}}(\mathbf{k}, z) = \left( \frac{H}{\dot{\phi}} \right)^2 \mathcal{P}_{\delta\phi_{\mathbf{k}}} = \frac{1}{2m_{pl}^2 \epsilon} \left( \frac{H}{2\pi} \right)^2 \left( \frac{k}{|k_{eff}|} \right)^3, \quad (37)$$

where  $m_{pl}$  is the Planck mass. The above equation could be parameterized as

$$\mathcal{P}_{\mathcal{R}}(\mathbf{k}, z) \equiv \mathcal{P}_{\mathcal{R}}^{iso}(k) \left( 1 - 3B + 6B^2 - \frac{3}{2} \left( \frac{3B'}{2k} \right)^2 (\hat{\mathbf{k}} \cdot \hat{\mathbf{n}})^2 \right), \quad (38)$$

where  $\mathcal{P}_{\mathcal{R}}^{iso}(k) \equiv A_{\mathcal{R}}^2 (k/H)^{n_{\mathcal{R}}-1}$ ,  $A_{\mathcal{R}}$  denotes the normalized amplitude and  $n_{\mathcal{R}}$  denotes the spectral index of the comoving curvature perturbation. We remain only the terms that are not higher than  $B^2(\mathbf{r} \cdot \hat{\mathbf{n}})$  orders. Here  $\hat{\mathbf{n}}$  denotes the third spatial direction and  $\mathbf{r}$  is the spatial location. The terms containing  $B$  induce the anisotropic behaviors of the primordial power spectra of the comoving curvature perturbation. The reason is that  $B = B(\mathbf{r} \cdot \hat{\mathbf{n}})$  depends on the third spatial coordinate  $z = \mathbf{r} \cdot \hat{\mathbf{n}}$ . This term would give arise to the (parity violating) statistical anisotropy of the universe. It was noteworthy that the level of the statistically anisotropic effects grows with the increase of the third spatial distance  $|z|$ . Thus, the statistical anisotropy would be significant at large scales. At small scales, however, the anisotropic effects are not significant in this model. Note that the above primordial power spectra (38) could be constrained by the Planck's results via a similar analysis in the reference [72, 73].

As an example, we refer to the linear approximation of  $B(z)$  in (21), i.e.,  $B(z) = f_1 z + o(z^2)$ . Here  $f_1$  is a constant coefficient with the dimension  $[length]^{-1}$ . Thus,  $B$  becomes

$$B(z) = f_1 \mathbf{r} \cdot \hat{\mathbf{n}}. \quad (39)$$

By substituting the above equation into (38), one could obtain the primordial power spectra as

$$\mathcal{P}_{\mathcal{R}}(\mathbf{k}, \mathbf{r}) \equiv \mathcal{P}_{\mathcal{R}}^{iso}(k) \left( 1 - 3f_1 \hat{\mathbf{n}} \cdot \mathbf{r} + 6f_1^2 (\hat{\mathbf{n}} \cdot \mathbf{r})^2 - \frac{27f_1^2}{8k^2} (\hat{\mathbf{n}} \cdot \hat{\mathbf{k}})^2 \right). \quad (40)$$

The last two terms refer to the quadrupole asymmetry in the above equation. Generically, they are smaller than the dipolar term. Thus, one could discard them if one is only interested in the dipolar modulation of the power spectra. In this way, the primordial power spectra (40) could be rewritten as

$$\mathcal{P}_{\mathcal{R}}(\mathbf{k}, \mathbf{r}) = \mathcal{P}_{\mathcal{R}}^{iso}(k) (1 - 3f_1 \hat{\mathbf{n}} \cdot \mathbf{r}). \quad (41)$$

This dipolar modulation of primordial power spectra could be used to account for the hemispherical asymmetry (and the parity asymmetry) of the CMB temperature-fluctuation amplitude observed by the WMAP and Planck satellites.

## VI. CONCLUSIONS AND DISCUSSIONS

In this paper, we have studied a generalized FRW metric with weak anisotropy in the Randers spacetime. The osculating Riemannian approach was employed to obtain the evolution of the Randers spacetime. We found an inflationary solution of Einstein's gravitational field equations at zero order. Meanwhile, we also obtained the anisotropic modifications to the inflationary phase of the very early universe. Most importantly, the primordial power spectra of the scalar perturbation (and the comoving curvature perturbation) was acquired by analysis on the equation of motion for the primordial fluctuations. It was found to be statistically anisotropic at large scales along the third spatial axis. This could reveal the deviations from the isotropy of the universe. The anisotropic effects are significant at large scales while they are insignificant at small scales in this model. This is consistent with the Planck 2013 results [2] released recently.

The primordial power spectra with direction dependence may account for certain anomalies of the CMB observations by the WMAP and Planck satellites [74–80]. In this paper, we obtained such a primordial power spectra with weak anisotropy in the Randers spacetime. Actually, the Randers spacetime has an extra vector field in the Randers structure. It could be related with certain (primordial) electromagnetic field with a privileged orientation [54]. It singles out a preferred direction in the spacetime background. This could influence the inflationary evolution of the very early universe and induce the asymmetry of the primordial power spectra. From the primordial power spectra of the scalar perturbation in this paper, we could see that the parity-odd direction-dependence (such as the dipolar modulation) may reveal the hemispherical power asymmetry and the parity asymmetry in the CMB data such as the WMAP and Planck. In addition, the parity-odd part has been pointed out to explain the alignment of low multipoles of the CMB recently [79, 80]. It is noteworthy that the Randers spacetime could also explain the large-scale bulk flow [47] and the anisotropic indications of the Hubble diagram [51].

As we have mentioned above, Finsler geometry is a reasonable candidate platform to

study the spacetime asymmetry and anisotropy. The Finsler spacetime is intrinsically anisotropic and direction dependent. The physical processes in Finsler spacetime should acquire similar anisotropic properties. In this paper, we indeed obtained the anisotropic effects on the inflation and the primordial power spectra from Finsler geometry. To our knowledge, this is the first time that the primordial power spectra was obtained in the Finsler geometric framework. Meanwhile, the statistically anisotropic properties were predicted for the primordial power spectra. Fortunately, the Planck's observations showed evidence for the deviations from isotropy. This is a good chance to test the anisotropic predictions of Finsler geometry. Inversely, Finsler geometry provides a natural explanation to the statistical anisotropy of the universe.

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